# Subset Construction: Algorithmic Specification

Given an *NFA* $N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$:

**ALGORITHM**: *ReachableSubsetStates*

**INPUT**: $q_0 : Q_N$ ; **OUTPUT**: $Reachable \subseteq \mathbb{P}(Q_N)$

**PROCEDURE**:

```
Reachable := { {q0} }
ToDiscover := { {q0} }
while ( ToDiscover ≠ ∅ ) {
    choose S : ℙ(Q_N) such that S ∈ ToDiscover
    remove S from ToDiscover
    NotYetDiscovered :=
        ( { {δ_N(s,0) | s ∈ S} } ∪ { {δ_N(s,1) | s ∈ S} } ) \ Reachable
    Reachable := Reachable ∪ NotYetDiscovered
    ToDiscover := ToDiscover ∪ NotYetDiscovered
}
return Reachable
```

| state \ input | 0 | 1 |
|---|---|---|
| $\{q_0\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_1\}$ | $\{q_0, q_2\}$ |
| $\{q_0, q_2\}$ | $\{q_0, q_1\}$ | $\{q_0\}$ |

# epsilon-NFA: Motivation

Draw NFA

$$\left\{ xy \;\middle|\; \begin{array}{ll} & x \in \{0,1\}^* \\ \wedge & y \in \{0,1\}^* \\ \wedge & x \texttt{ has alternating } 0'\texttt{s and } 1'\texttt{s} \\ \wedge & y \texttt{ has an odd \# } 0'\texttt{s and an odd \# } 1'\texttt{s} \end{array} \right\}$$

$$\left\{ w : \{0,1\}^* \;\middle|\; \begin{array}{ll} & w \texttt{ has alternating } 0'\texttt{s and } 1'\texttt{s} \\ \vee & w \texttt{ has an odd \# } 0'\texttt{s and an odd \# } 1'\texttt{s} \end{array} \right\}$$

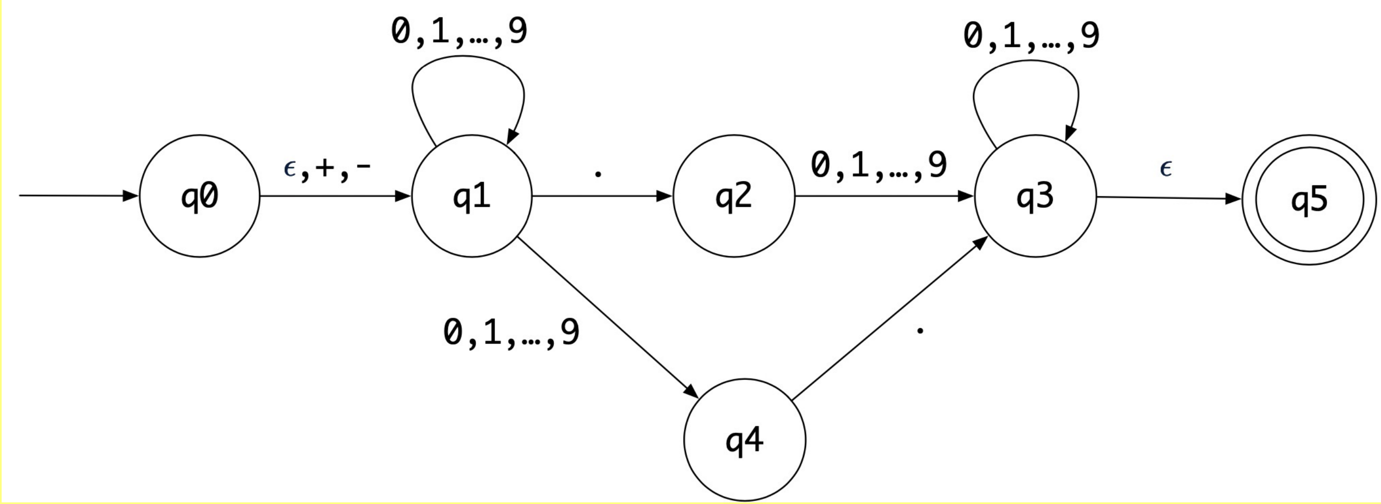
# epsilon-NFA: Example

$$\left\{ sx.y \;\middle|\; \begin{array}{ll} & s \in \{+, -, \epsilon\} \\ \wedge & x \in \Sigma_{dec}^{*} \\ \wedge & y \in \Sigma_{dec}^{*} \\ \wedge & \neg(x = \epsilon \wedge y = \epsilon) \end{array} \right\}$$

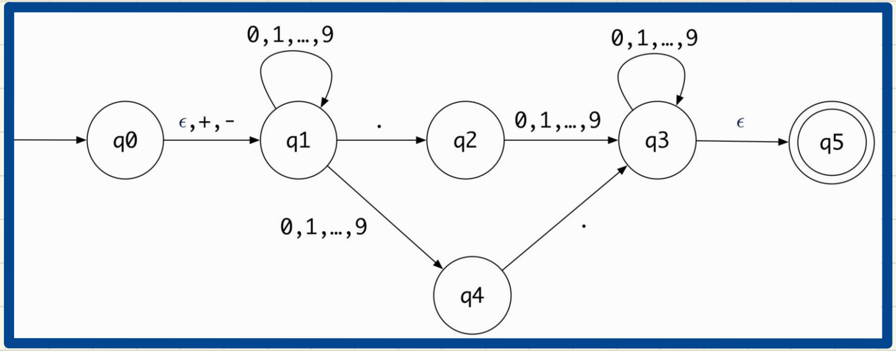
Is this a DFA?

Is this an NFA?

Is this an ε-NFA?



# epsilon-NFA: Formulation (1)




An *ϵ-NFA* is a 5-tuple

$$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$$

**Draw the transition table.**

| | $\epsilon$ | +, - | . | 0 .. 9 |
|---|---|---|---|---|
| $q_0$ | | $\{q_1\}$ | $\varnothing$ | $\varnothing$ |
| $q_1$ | | $\varnothing$ | $\{q_2\}$ | $\{q_1, q_4\}$ |
| $q_2$ | | $\varnothing$ | $\varnothing$ | $\{q_3\}$ |
| $q_3$ | | $\varnothing$ | $\varnothing$ | $\{q_3\}$ |
| $q_4$ | | $\varnothing$ | $\{q_3\}$ | $\varnothing$ |
| $q_5$ | | $\varnothing$ | $\varnothing$ | $\varnothing$ |

# Regular Expression to epsilon-NFA

| Base Cases | Recursive Cases (given REs E and F) |
| --- | --- |
| $\varepsilon$ | $R + S$ |
| $\varnothing$ | $RS$ |
| $a \quad (a \in \Sigma)$ | $R^*$ |

# Regular Expression to epsilon-NFA: Example

$(0 + 1)^*1(0+1)$